

## Section Handout 5

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### Problem One: Designing PDAs

Below are a list of alphabets and languages over those alphabets. For each language, design a pushdown automaton that recognizes the given language.

- i. Let  $\Sigma = \{0, 1, ?\}$  and let  $L = \{x?y \mid x, y \in \{0, 1\}^* \text{ and } y \text{ is the reverse of } x\}$ . Design a **deterministic** PDA that recognizes  $L$ .
- ii. Let  $\Sigma = \{0, 1, 2\}$  and let  $L = \{0^m 1^n 2^p \mid m, n, p \in \mathbb{N} \wedge (m = n \vee m = p)\}$ . Design a PDA that recognizes  $L$ .

### Problem Two: The Pumping Lemma

Let  $\Sigma = \{0, 1, \mathbf{A}, \mathbf{B}\}$  and let  $TWOWAYBALANCE = \{w \mid w \text{ contains the same number of } 0\text{s and } 1\text{s and the same number of } \mathbf{A}\text{s and } \mathbf{B}\text{s}\}$ . Prove that  $TWOWAYBALANCE$  is not context-free.\*

### Problem Three: Turing Machine Design

Write a **WB** program that accepts the language of all balanced parentheses over  $\Sigma = \{ (, ) \}$ . You can assume that the input has been shifted over by one step, leaving a blank cell in the first position on the tape, and that the tape head is now reading the very first symbol of the input (or a blank if the input was empty).

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\* This problem adapted from Problem 2.32 from Sipser.